SEPARATION OF FORWARD AND BACKWARD ACOUSTIC WAVES IN A CAR EXHAUST BY ARRAY PROCESSING*

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ABSTRACT

We present in this paper a new interdisciplinary application of digital signal processing and its result in comparison with the application of classical methods. The problem is the separation of two acoustic waves propagating along an exhaust pipe. It has been proved that the use of the array signal processing technique - signals are registered by an array of sensors placed along the pipe and processed by means of the LCMV beamformer - has achieved very good results under bad conditions as the uncertainty in the speed propagation knowledge.

The array technique has also been applied to the separation of two pressure waves generated by the discharge of a fuel injection pump. In this case results have been good but a preprocessing must be done due to the pulse character of these pressure waveforms.

1. INTRODUCTION

There are two low frequency wideband acoustic waves propagating along a car exhaust when the engine is working, the forward one generated by the engine, and the backward one originated from the maladjustment of acoustic impedance at the muffler input. Both waves are cyclic and their waveforms depend on the number of revolutions-per-minute the car engine is rotating at.

To separate the excitation signal from the reflected one at the muffler input is an essential procedure in order to analyze the muffler features. The separation is specifically needed when the reflection coefficient and the transmission loss have to be estimated. This estimate is done by techniques of system identification, i.e., the system input and output must be known [1].

The decomposition of the waves in the car exhaust is also useful for carrying out two purposes in active control of noise methods: to avoid the acoustic feedback of the canceler signal to the reference sensor and to measure the actual output wave at the pipe end without the superposition of the backward signal.

The proposed technique relies on placing a certain number of sensors at a suitable location along the exhaust pipe where the separation of the forward wave from the backward one is wanted. In contrast to the traditional techniques that need a thermodynamic model of the acoustic waves propagation [2], our technique is based on array signal processing.

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2. PROPOSED TECHNIQUE

When it comes to deciding which array processing technique would fit the problem, it is necessary to point out the most important characteristics of the experiment we are going to work with. These conditions are:

1. The acoustic waves are broadband signals (typically from 0 to 2000 Hz).
2. The forward and backward components of the signal measured by the sensors have well known angles of arrival: respectively -90° and 90° with respect to the orthogonal direction of the pipe.
3. The number of sensors must be small (between 2 and 4 sensors) by restrictions of the data acquisition system. As we have said, they are placed along the pipe so the array will be linear.
4. The propagation speed of the waves is uncertainly known because its value depends on several parameters, mainly the temperature, the rotating speed of the engine and the acoustic pressure.
5. The pressure level of the acoustic waves can be very large. In this case the linear superposition of the forward and backward waveforms does not hold and the sum of both waves is given by the following expression [3]:

$$\left[\frac{P_m}{P_0}\right]^{\frac{\gamma-1}{2\gamma}} = \left[\frac{P_{for}}{P_0}\right]^{\frac{\gamma-1}{2\gamma}} + \left[\frac{P_{back}}{P_0}\right]^{\frac{\gamma-1}{2\gamma}} - 1 \quad (1)$$

where P_0 is the reference pressure and it has a value of 1 bar, γ is the ratio of the specific heats (typically 1.35) and P_m, P_{for} and P_{back} are the measured pressure at the sensors, the forward wave pressure and the backward wave one respectively.

Once the characteristics of the problem have been stated, we decided to use the broadband *Linearly Constrained Minimum Variance* beamformer algorithm [4][5] in order to separate the waves. The basis of the LCMV technique is the aim of the following section.

2.1. The LCMV Beamformer

The goal of an LCMV beamformer is to generate an spatial response that minimize the output signal variance, joined to constraining the beamformer response to some predetermined directions of arrival. We will analyze the procedure in the following paragraphs.

The linear array response corresponding to a wave of frequency f propagating from direction θ is expressed by

$$\mathbf{H}(\theta,f) = \mathbf{w}^{+} \cdot \mathbf{d}(\theta,f) \qquad (2)$$

where $\mathbf{w}^{+}$ is the weighting vector transposed and conjugated, and $\mathbf{d}$ is the array steering vector whose expression is

$$\mathbf{d}(\theta,f) = [\ 1\ \ e^{-j2\pi f \frac{d_2 \,\mathrm{sen}(\theta)}{c}} \cdots e^{j2\pi f \frac{d_N \,\mathrm{sen}(\theta)}{c}}\ ]^{T} \qquad (3)$$

where d_i is the distance between the i-th sensor and the first one, N is the number of sensors and c is the propagation speed.

If we impose the following constraint to the weight vector

$$\mathbf{w}^{+}\ \mathbf{d}(\theta_0,f_0) = g \qquad (4)$$

where g is a complex constant, we will force the beamformer to give a response g to every signal of frequency f_0 coming from direction θ_0. In addition to (4), it is possible to minimize the contribution of the interfering signals at the beamformer output by the appropriate choice of the weights vector. For this purpose we need the mean power (or the mean squared value) of the output signal to be minimum.

Therefore the choice of the weights vector $\mathbf{w}$ would arise from constraining the beamformer to satisfy both conditions: the minimization of the output power

$$E\left[|y|^2\right] = \mathbf{w}^{+} \cdot \mathbf{R}_x \cdot \mathbf{w} \qquad (5)$$

subject to

$$\mathbf{d}(\theta,f)^{+} \cdot \mathbf{w} = g^{*} \qquad (6)$$

where $\mathbf{R}_x$ is the autocorrelation function of the input signal x(t).

The Lagrange method is used to solve the equations (5) and (6) and the solution is [6]

$$\mathbf{w} = g^{*} \frac{\mathbf{R}_x^{-1} \cdot \mathbf{d}(\theta,f)}{\mathbf{d}(\theta,f)^{+} \cdot \mathbf{R}_x^{-1} \cdot \mathbf{d}(\theta,f)} \qquad (7)$$

In order to fix the array response to different directions and frequencies, we can impose more lineal constraints. For example, if we want to eliminate a non-desired signal propagating from direction ϕ, we can force the beamformer to have a null response at that direction. Together with (4), this constraint would be expressed as

$$\begin{bmatrix} \mathbf{d}(\theta,f)^{+} \\ \mathbf{d}(\phi,f)^{+} \end{bmatrix} \cdot \mathbf{w} = \begin{bmatrix} g^{*} \\ 0 \end{bmatrix} \qquad (8)$$

If there are L<N linear constraints over $\mathbf{w}$, we have

$$\mathbf{C}^{+} \cdot \mathbf{w} = \mathbf{g} \qquad (9)$$

where $\mathbf{C}$ (NxL) is called the constraint matrix and $\mathbf{g}$ (Lx1) is called the response vector. If we suppose that the constraints are linearly independent, then the optimum solution to (5) subject to (6) is

$$\mathbf{w} = \mathbf{R}_x^{-1} \cdot \mathbf{C} \cdot \left[\mathbf{C}^{+} \cdot \mathbf{R}_x^{-1} \cdot \mathbf{C}\right]^{-1} \cdot \mathbf{g} \qquad (10)$$

Finally, the output signal of the beamformer, y, can be estimated by

$$y = \mathbf{w}^{+} \cdot \mathbf{x} \qquad (11)$$

3. APPLICATION OF LCMV TO THE SEPARATION OF ACOUSTIC WAVES

In the previous point we have presented the procedure to build an LCMV narrowband beamformer. In our case, the signals of interest are broadband, so the previous algorithm has been reformulated in a broadband version [5]:

1. We calculate the FFT of the broadband signals registered in every sensor.
2. We take the i-th frequency component of every signal and apply the LCMV narrowband beamformer to this frequency in order to obtain the beamformer output for every frequency and for the desired direction of arrival.
3. We calculate the inverse FFT to obtain the output waveforms.

Therefore, the only distinction between the narrowband and the broadband application is that the snapshot vector changes for every frequency and the waves spectra are estimated at every frequency independently of the rest of frequencies. If N is the number of sensors, the snapshot is made up for the following components

$$\mathbf{x}(\omega) = \left[X_1(\omega)\ X_2(\omega) \cdots X_N(\omega)\right]^{T} \qquad (12)$$

where $X_i(\omega)$ is the value at frequency ω of the FFT of $x_i(t)$, and $x_i(t)$ is the registered pressure at the i-th sensor.

In our case, there are only two signals propagating along the pipe. We know for both of them the direction of arrival (θ = -90° for the forward wave and θ = 90° for the backward one) and their propagation speed can be estimated.

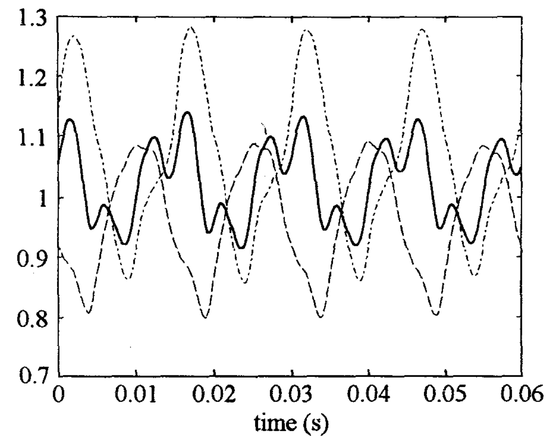

Fig.1: Forward (-.), backward (--) and sum wave (-) along the exhaust for an engine rotating at 2000 r.p.m..

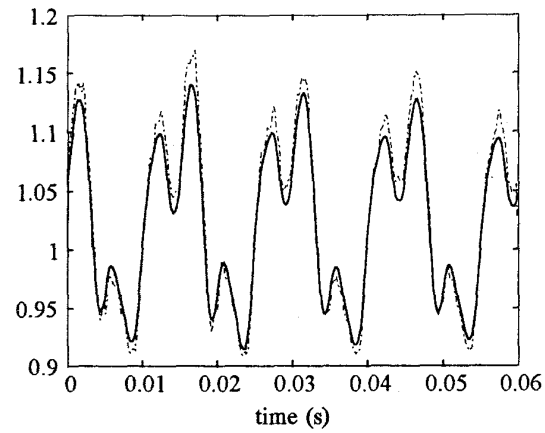

Fig.2: Measured (-.) and sum wave (-) along the exhaust for an engine rotating at 2000 r.p.m..

Therefore, by means of (2) we should generate the direction vectors for the constraint matrix **C** and we calculate **w** using (10). If we state

$$\mathbf{C} = \begin{bmatrix} \mathbf{d}(\theta = -90°, f)^{+} \\ \mathbf{d}(\theta = 90°, f)^{+} \end{bmatrix} \quad (13)$$

the forward wave can be obtained imposing the response vector $\mathbf{g} = [1\ 0]^T$ in (10) and the backward one is obtained imposing $\mathbf{g} = [0\ 1]^T$.

Figures 1 and 2 show the forward and backward waves and the sum of both signals in comparison with the measured one. In this case the sensor array had 4 elements 5 cm apart one from another, the car engine was rotating at 2000 r.p.m. and the estimated propagation speed was 623 m/s. Signals were sampled at a frequency rate of 8,533 Hz.

The verification of the results showed in the figures 1 and 2 is carried out through the comparison with the classic method results. Conclusions are left to the last point of the paper.

4. OTHER APPLICATIONS: PRESSURE WAVES

Once we have stated a suitable digital array processing technique to separate the acoustic waves propagating along a car exhaust, we will test the broadband LCMV beamformer with the pressure waves generated by the gas emerging from a fuel injection pump.

The problem is similar to the acoustic waves since there are also two fluid waves propagating along the pipe. However, the signal features are quite different just as set out below:

1. The signal is not cyclic but the waveform is a pulse of pressure with a sharp rise and a transitory tail that returns at the end to the rest position. Therefore, this kind of signals can not be considered stationary.
2. The bandwidth is larger than in the engine noise case. Particularly, its value stands in the range of 3 to 10 Khz depending on the number of cycles per minute the injection pump is discharging at.
3. The propagation speed of the signal is known more precisely than in the acoustic waves because the temperature is well controlled during the process.

In this case we have to deal with two main problems. First, the signal registration at the sensors consists in three time intervals: a rest interval when there is no signal, the pulse interval and the third one when the signal is returning to rest till there is no signal again. The separation of the waves is not possible in the rest interval (there is no wave) so the technique fails if this part is not previously suppressed.

Secondly, the calculation of the FFT implies a cyclic repetition of the pulse. In our case, it is difficult to assure the continuity of the beginning and the end of the registered pulse in order to simulate a cyclic signal, so the separation of the waves does not fit exactly at the extremes.

Although the previous problems are quite critical, the algorithm has been proved to give good results when the no-signal intervals have been cut off and the first and last points approximately keep the continuity of the cyclic repetition. Some results for an injection pump discharging at 750 cycles/min. and 4.5 mgr./cycle are shown in figures 3 and 4. The waves are propagating at 1400 m/s and were sampled at a rate of 32,143 Hz. The array were composed by two sensors placed 8.2 cm apart.

5. NEW CONTRIBUTIONS OF THE PROPOSED TECHNIQUE

It must be pointed out that it is the first time that the array signal processing is applied to the separation of acoustic, and pressure waves in general, propagating along a pipe. Therefore this application is a step forward in the insertion of

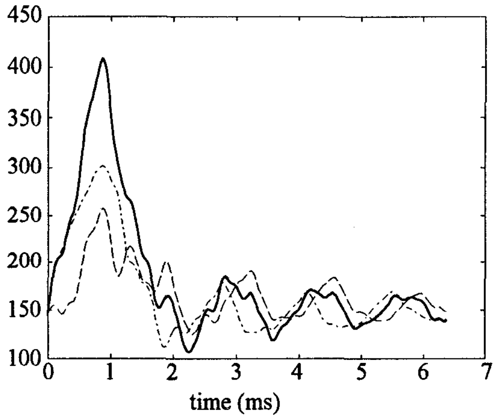

Fig. 3: Forward (-.), backward (--) and sum wave (-) along the pipe for an injection pump.

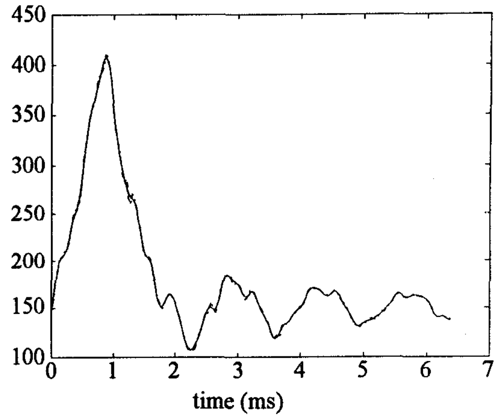

Fig. 4: Measured (-.) and sum wave (-) along the pipe for an injection pump.

digital signal processing in disciplines which were traditionally unaware of the DSP power.

In the case of separating the acoustic waves in an exhaust pipe, the classic method relies on a thermodynamic model of the propagating waves and its main limitations are the following:

- The separation accuracy depends on the good estimate of two parameters: the temperature and the propagation speed of the forward and backward waves. This estimate can not be reliably done because the propagation speed in his turn depends on the temperature and the pressure level, which are varying with time.
- The classic algorithm is able to separate the waves up to a maximum frequency of 1 KHz approximately. However, the muffler features, such as the reflection coefficient and the transmission loss, must be identified at least in the 2 KHz frequency range.

In contrast to these limitations, the separation of waves by means of array processing relies on the knowledge of three parameters: the distance between sensors, the direction of arrival and the propagation speed of the signals. The first two data are precisely known in our experiment whereas the third one is more or less uncertain.

However, this uncertainty has appeared not to be critical in the acoustic case since the final results have been satisfactory inside a range of variability of 40% around the estimated value of the propagation speed.

In addition to the improvement in the independence of the propagation speed knowledge, our technique can estimate the forward and backward waves spectra up to a maximum frequency limited only by the distance between sensors (it must be small enough to avoid the spatial aliasing). Therefore the knowledge of the muffler excitation is assured over the 2 KHz frequency.

Finally we would like to point out that the array processing algorithm has also been used in the separation of waves whose waveforms are not cyclic as it is the output pressure of a fuel injection pump. This wave is an impulse so there is an extra condition: the decomposition of the waves does not work during the time there is no signal in the pipe. Once we have eliminated the samples where the signals are not present and we have obeyed the cyclic constraints of the FFT (the measured signal beginning and end values must be similar) then we have also obtained good results in the decomposition of waves.

Therefore, we can conclude that the spatial filtering has proved to be efficient in its application to acoustic waves and in general to pressure waves since we can obtain with this technique robust and good results.

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