Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 283 (2005) 459–466
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

Acoustic attenuation performance analysis of multi-chamber reactive silencers

Z.L. Ji

School of Power and Nuclear Energy Engineering, Harbin Engineering University, Harbin, Heilongjiang 150001, P.R. China

Received 29 August 2003; received in revised form 21 April 2004; accepted 13 May 2004

1. Introduction

The multi-chamber reactive silencers are widely used to reduce engine exhaust noise as well as blower intake and discharge noise. Davis et al. [1] developed a one-dimensional analytical approach, which assumes plane wave propagation in the axial direction in silencers, to predict the acoustic attenuation performance of double expansion chamber reactive silencers, and compared the predictions with experimental results. They investigated the effect of the length of inter-connecting tube on the acoustic attenuation performance of the silences with chambers of equal lengths. The transfer matrix approach based on the one-dimensional plane wave theory is described in detail by Munjal [2] for several acoustic elements and silencer configurations. The plane wave theory, however, excludes the effect of multi-dimensional waves inside the silencers. Thus, while yielding reasonable predictions at lower frequencies, the simplistic approach is expected to deviate from experimental results at higher frequencies. Above the plane wave cut-off frequency, one-dimensional plane wave approach is not applicable to predict the acoustic attenuation performance of silencers. For the large size silencers, the plane wave cut-off frequency is lower and high-frequency acoustic attenuation performance prediction is still desired for the purpose of noise control. Therefore, a multi-dimensional approach is required for the accurate prediction of acoustic attenuation performance of silencers. The boundary element method (BEM) is suitable to predict the acoustic attenuation performance of reactive silencers with any

E-mail address: zhenlinji@yahoo.com (Z.L. Ji).

doi:10.1016/j.jsv.2004.05.013

given geometry [3–5], and they are clearly no longer confined to a plane wave treatment. The BEM predictions agree very well with experiments and three-dimensional analytical results for several silencer configurations [6–8].

The objective of the present study is (1) to develop a numerical approach based on the three-dimensional substructure boundary element technique to predict the acoustic attenuation performance of multi-chamber reactive silencers with inter-connecting tube(s); (2) to examine and compare the acoustic attenuation characteristics of single, double and triple expansion chamber silencers; (3) to investigate the effect of internal geometry, including length of inter-connecting tube, location of bulkhead, extensions of inlet and outlet tubes into chambers, as well as the number of inter-connecting tubes, on the acoustic attenuation performance of the silencers; and (4) to explore the optimal design of the multi-chamber reactive silencers. The results of this study provide a clear insight into understanding the acoustic attenuation characteristics of multi-chamber reactive silencers.

2. Formulation

To employ the boundary element method for the prediction of the acoustic attenuation performance of multi-chamber reactive silencers with inter-connecting tube(s), a multidomain approach is needed due to the presence of singular boundaries [4,5]. The silencer considered here is divided into a few substructures as shown in Fig. 1: inlet tube, expansion chambers,

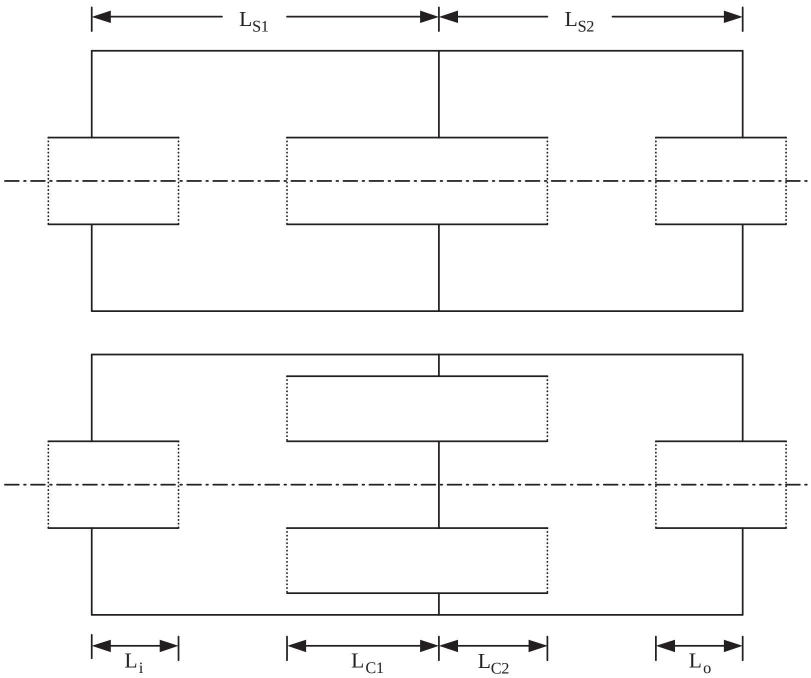

Fig. 1. Double expansion chamber silencers with single and double inter-connecting tubes.

inter-connecting tube(s), and outlet tube. For each substructure, the BEM gives [5]

$$\left[H^{S_j}\right]\left\{P^{S_j}\right\} = \rho_0 c_0 \left[G^{S_j}\right]\left\{V^{S_j}\right\}, \tag{1}$$

where $\left[H^{S_j}\right]$ and $\left[G^{S_j}\right]$ are the coefficient matrices; $\left\{P^{S_j}\right\}$ and $\left\{V^{S_j}\right\}$ are the vectors of acoustic pressure and outward normal particle velocity at boundary nodes, respectively, for the substructure j; $\rho_0 c_0$ is the characteristic impedance of the medium. The boundaries are grouped into the inlet, outlet and rigid wall represented by the subscripts i, o and w, respectively. The objective of the following derivation is to develop the relationship between inlet variables (P_i, V_i) and outlet variables (P_o, V_o) which then facilitates the calculation of the four-pole parameters and transmission loss. Eq. (1) combined with the rigid wall boundary condition $V_w = 0$, yields

$$\left\{ \begin{array}{c} P_i^{S_j} \\ P_o^{S_j} \end{array} \right\} = \rho_0 c_0 \begin{bmatrix} T_{11}^{S_j} & T_{12}^{S_j} \\ T_{21}^{S_j} & T_{22}^{S_j} \end{bmatrix} \left\{ \begin{array}{c} V_i^{S_j} \\ V_o^{S_j} \end{array} \right\}. \tag{2}$$

For two substructures m and n in series, the following relationship may be derived by the continuity conditions of acoustic pressure and particle velocity on the interface as

$$\left\{ \begin{array}{c} P_i^{S_m} \\ P_o^{S_n} \end{array} \right\} = \rho_0 c_0 \left[T_S^R\right] \left\{ \begin{array}{c} V_i^{S_m} \\ V_o^{S_n} \end{array} \right\}, \tag{3}$$

where

$$\left[T_S^R\right] = \begin{bmatrix} T_{11}^{S_m} - T_{12}^{S_m}(T_{11}^{S_n} + T_{22}^{S_m})^{-1} T_{21}^{S_m} & T_{12}^{S_m}(T_{11}^{S_n} + T_{22}^{S_m})^{-1} T_{12}^{S_n} \\ T_{21}^{S_n}(T_{11}^{S_n} + T_{22}^{S_m})^{-1} T_{12}^{S_m} & T_{22}^{S_n} - T_{21}^{S_n}(T_{11}^{S_n} + T_{22}^{S_m})^{-1} T_{12}^{S_n} \end{bmatrix} \tag{4}$$

is the resultant impedance matrix for the substructures m and n in series.

For two substructures m and n in parallel (for example, the dual inter-connecting tubes), the acoustic pressure and outward normal particle velocity at inlet and outlet nodes may be expressed as

$$\left\{ \begin{array}{c} P_i^{S_m} \\ P_i^{S_n} \\ P_o^{S_m} \\ P_o^{S_m} \end{array} \right\} = \rho_0 c_0 \left[T_P^R\right] \left\{ \begin{array}{c} V_i^{S_m} \\ V_i^{S_n} \\ V_o^{S_m} \\ V_o^{S_m} \end{array} \right\}, \tag{5}$$

where the resultant impedance matrix

$$\left[T_P^R\right] = \begin{bmatrix} T_{11}^{S_m} & 0 & T_{12}^{S_m} & 0 \\ 0 & T_{11}^{S_n} & 0 & T_{12}^{S_n} \\ T_{21}^{S_m} & 0 & T_{22}^{S_m} & 0 \\ 0 & T_{21}^{S_n} & 0 & T_{22}^{S_n} \end{bmatrix}. \tag{6}$$

Finally, combining all substructures yields

$$\left\{ \begin{array}{c} P_i \\ P_o \end{array} \right\} = \rho_0 c_0 \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \left\{ \begin{array}{c} V_i \\ V_o \end{array} \right\}, \tag{7}$$

which defines the impedance matrix between the inlet and outlet of the silencer. Therefore, the transfer matrix can be obtained from the resultant impedance matrix for the silencer and represented as

$$\left\{ \begin{array}{c} \bar{p}_i \\ \rho_0 c_0 \bar{v}_i \end{array} \right\} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \left\{ \begin{array}{c} \bar{p}_o \\ \rho_0 c_0 \bar{v}_o \end{array} \right\}, \tag{8}$$

where $\bar{p}_i$, $\bar{p}_o$, and $\bar{v}_i$, $\bar{v}_o$ are the average acoustic pressures and particle velocities on the inlet and outlet, respectively; A, B, C and D are the four-pole parameters. The transmission loss of the silencers can be determined by

$$\mathrm{TL} = 20\log_{10}\{|A + B + C + D|/2\} + 10\log_{10}(S_i/S_o) \quad (\mathrm{dB}), \tag{9}$$

where S_i and S_o are the cross-sectional areas of the inlet and outlet of the silencer, respectively.

3. Results and discussion

For all configurations, the present study considers $d = 6''$ and $D = 18''$ for the inlet/outlet tube and silencer diameters, respectively, and speed of sound $c = 344\,\mathrm{m/s}$.

Fig. 2 shows the transmission loss comparison of single, double and triple expansion chamber silencers with equal length (24″) for each chamber. The double chamber silencer provides much higher acoustic attenuation than the single chamber silencer and the triple chamber silencer exhibits a higher acoustic attenuation than the double chamber silencer at most frequencies in the plane wave region. At higher frequencies, the acoustic attenuation of the multiple chamber silencers is slight higher than the single chamber silencer, but the acoustic behavior is complex. An

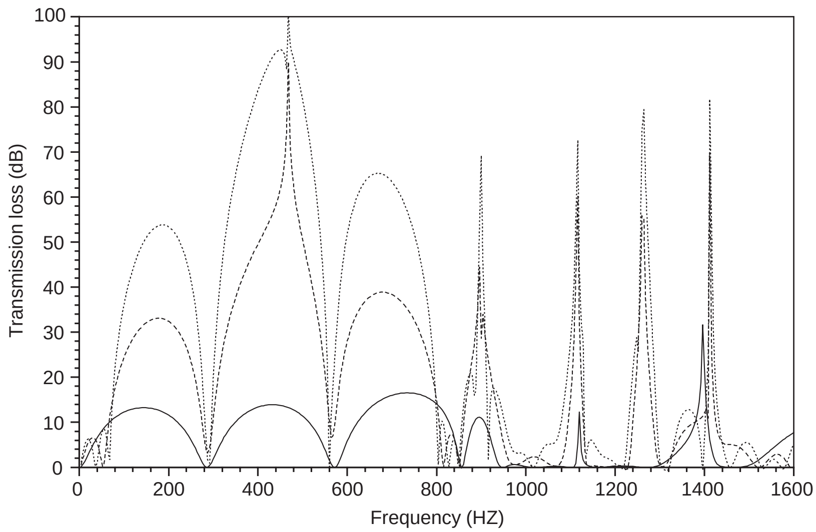

Fig. 2. Acoustic attenuation performance comparison of single and multiple expansion chamber silencers: ————, single chamber silencer (Ls = 24″); – – – – –, double chamber silencer (Ls1 = Ls2 = 24″, Lc1 = Lc2 = 6″); ...., triple chamber silencer (Ls1 = Ls2 = Ls3 = 24″, Lc1 = Lc2 = Lc3 = Lc4 = 6″).

observation has been found that the double chamber silencer produces a very low attenuation dome with a pass frequency, while triple chamber silencer presents two very low attenuation domes with two pass frequencies in the plane wave region. Fig. 3 compares the transmission loss of single, double and triple expansion chamber silencers, which have same overall length (48″). Similarly, the multiple chamber silencers show higher acoustic attenuation than the single chamber silencer at most frequencies, especially in the plane wave region. However, at very low frequencies, the acoustic attenuation of the double chamber silencer is lower than the single chamber silencer before the pass frequency, and the acoustic attenuation of the triple chamber silencer is lower than the double chamber silencer before the second pass frequency.

The double expansion chamber silencer with fixed length ($L_S = 48''$) is selected hereafter to investigate the effect of internal geometry on acoustic attenuation performance. The effect of length of inter-connecting tube on acoustic attenuation performance of the silencer is illustrated in Fig. 4. Increasing the length of inter-connecting tube shifts the resonance peaks and pass frequency to lower frequencies, while the troughs located at $kL_{S1} = kL_{S2} = n\pi$ were not changed in the plane wave region. The effect of bulkhead location (or chamber lengths) on acoustic attenuation characteristics of the silencer is shown in Fig. 5. For the fixed length of the inter-connecting tube, the pass frequency is fixed, but the trough locations are changed accordingly. Longer extension of inter-connecting tube into a chamber leads to a lower resonance frequency.

The effects of extensions of inlet and outlet tubes into chambers on acoustic attenuation performance of the silencer are analyzed next. Fig. 6 compares the transmission loss results of the silencer with and without inlet/outlet extensions. The extensions of inlet and outlet tubes into chambers contributed resonance peaks that may improve the acoustic attenuation in specific frequency ranges. By choosing the lengths of extended inlet/outlet tubes and inter-connecting tube to match the resonances with troughs of the silencers without extensions an excellent acoustic

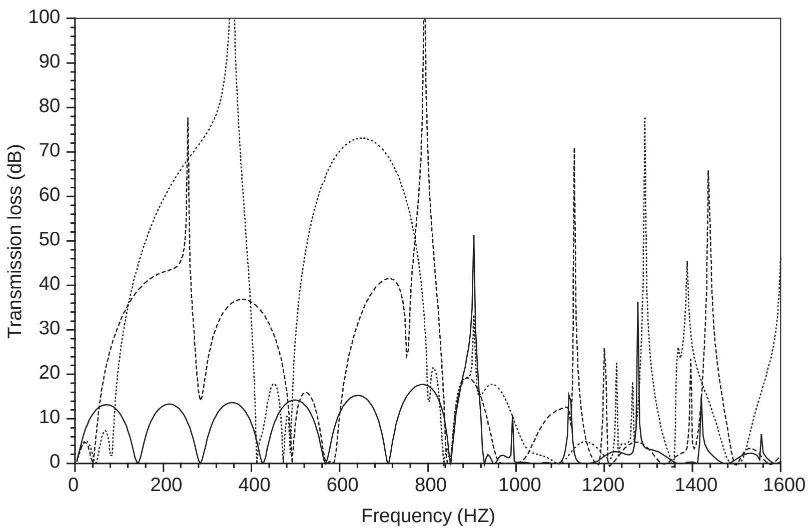

Fig. 3. Acoustic attenuation performance comparison of single and multiple expansion chamber silencers: ————, single chamber silencer (Ls = 48″); – – – – –, double chamber silencer (Ls1 = Ls2 = 24″, Lc1 = Lc2 = 12″); ...., Triple chamber silencer (Ls1 = Ls2 = Ls3 = 16″, Lc1 = Lc4 = 8″, Lc2 = Lc3 = 4″).

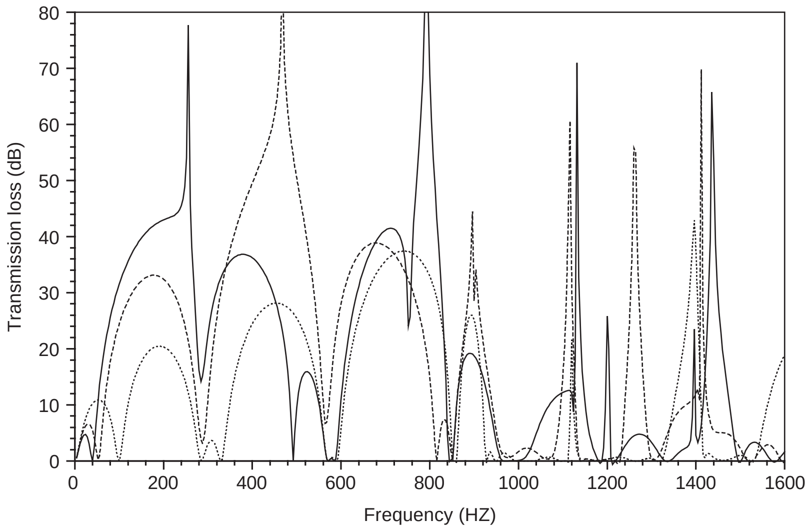

Fig. 4. Effect of length of inter-connecting tube on acoustic attenuation performance of double chamber silencer (Ls1 = Ls2 = 24″): ————, Lc1 = Lc2 = 12″; – – – – –, Lc1 = Lc2 = 6″; ...., Lc1 = Lc2 = 0.

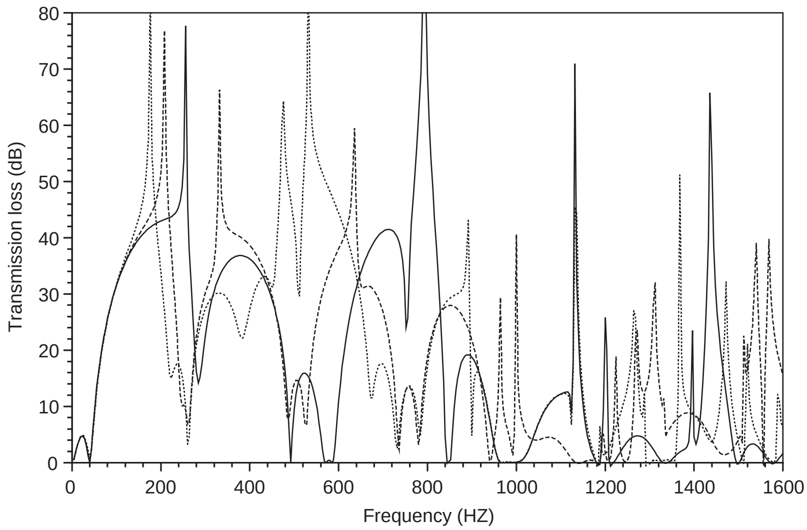

Fig. 5. Effect of location of bulkhead on acoustic attenuation performance of double chamber silencer: ————, Ls1 = Ls2 = 24″, Lc1 = Lc2 = 12″; – – – – –, Ls1 = 27″, Ls2 = 21″, Lc1 = 15″, Lc2 = 9″; ...., Ls1 = 30″, Ls2 = 18″, Lc1 = 18″, Lc2 = 6″.

attenuation may be obtained. Fig. 6 includes also transmission loss of an optimum designed silencer.

Finally, the number of inter-connecting tubes on acoustic attenuation performance of the double chamber silencer is examined. Fig. 7 compares the transmission loss of the silencers with

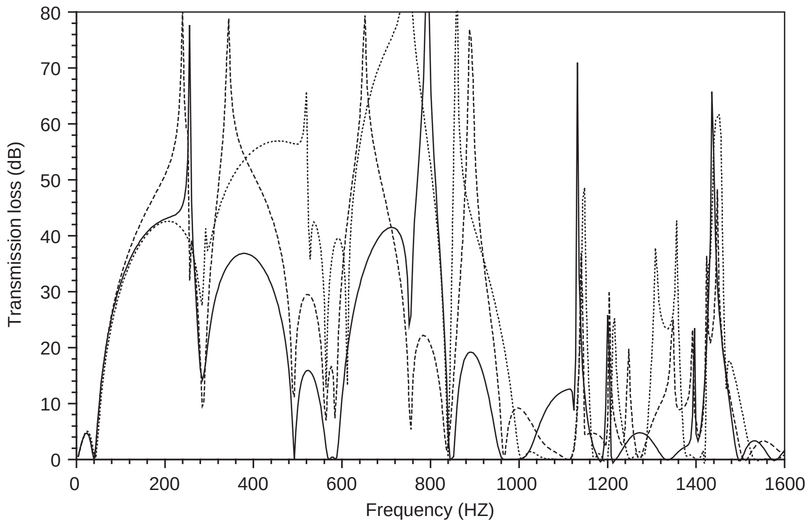

Fig. 6. Effect of extensions of inlet and outlet tubes into chambers on acoustic attenuation performance of double chamber silencer (Ls1 = Ls2 = 24″): ————, Lc1 = Lc2 = 12″, Li = Lo = 0; – – – – –, Lc1 = Lc2 = 12″, Li = 7″, Lo = 4″; ...., Lc1 = Lc2 = 10.5″, Li = 5″, Lo = 4″.

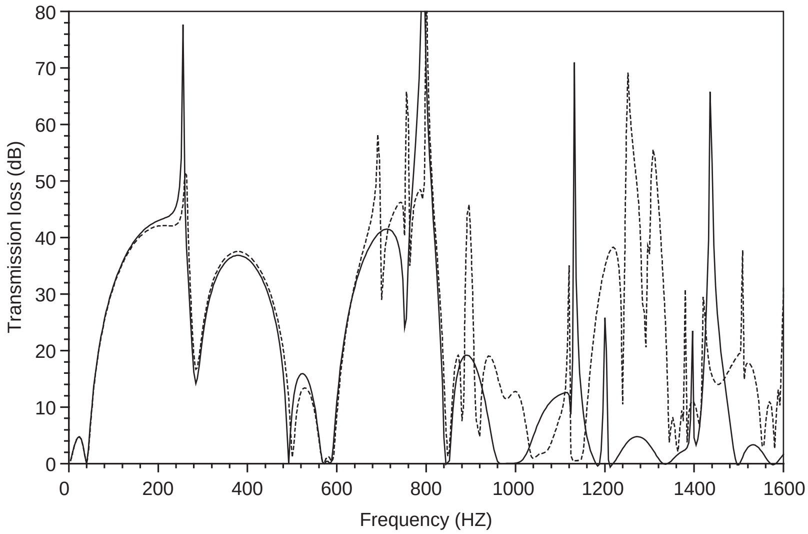

Fig. 7. Effect of the number of inter-connecting tube on acoustic attenuation performance of double chamber silencer (Ls1 = Ls2 = 24″, Lc1 = Lc2 = 12″): ————, single inter-connecting tube; ...., double inter-connecting tubes.

single co-axial inter-connecting tube and double offset inter-connecting tubes (5″ offset from the centerline), which have same total cross-sectional area. Both the silencers exhibit a very close acoustic attenuation at lower frequencies, while using double inter-connecting tubes improved the acoustic attenuation at higher frequencies.

4. Conclusions

A numerical approach based on the three-dimensional substructure boundary element technique is developed to predict and analyze the acoustic attenuation performance of multi-chamber reactive silencers with inter-connecting tube(s). The effect of geometry on acoustic attenuation performance of the silencers is investigated in detail. In general, the multiple chamber silencers provide higher acoustic attenuation than the single chamber silencer at most frequencies after their pass frequencies. Below their pass frequencies, the use of multiple chambers lowers the acoustic attenuation compared to the single chamber silencer with the same silencer length. Reasonably, choosing the lengths of extended inlet/outlet tubes and inter-connecting tube may obtain a desired broadband high noise attenuation characteristics. The dual inter-connecting tubes improve the acoustic attenuation performance of the silencer at higher frequencies.

References

[1] D.D. Davis, G.M. Stokes, D. Moore, G.L. Stevens, Theoretical and experimental investigations of mufflers with comments on engine exhaust muffler design, NACA TN 1192, 1954.

[2] M.L. Munjal, *Acoustics of Ducts and Mufflers*, Wiley-Interscience, New York, 1987.

[3] A.F. Seybert, C.Y.R. Cheng, Application of the boundary element method to acoustic cavity response and muffler analysis, *Journal of Vibration, Acoustics, Stress, and Reliability in Design* 109 (1987) 15–21.

[4] C.Y.R. Cheng, A.F. Seybert, A multidomain boundary element solution for silencer and muffler performance prediction, *Journal of Sound and Vibration* 151 (1991) 119–129.

[5] Z.L. Ji, Q. Ma, Z.H. Zhang, Application of the boundary element method to predicting acoustic performance of expansion chamber mufflers with mean flow, *Journal of Sound and Vibration* 173 (1994) 57–71.

[6] A. Selamet, Z.L. Ji, P.M. Radavich, Acoustic attenuation performance of circular expansion chambers with offset inlet/outlet: II. Comparison with computational and experimental studies, *Journal of Sound and Vibration* 213 (1998) 619–641.

[7] A. Selamet, Z.L. Ji, Acoustic attenuation performance of circular expansion chambers with extended inlet/outlet, *Journal of Sound and Vibration* 223 (1999) 197–212.

[8] Z.L. Ji, A. Selamet, Boundary element analysis of three-pass perforated duct mufflers, *Noise Control Engineering Journal* 48 (2000) 151–156.